The Einstein field equation in terms of the Schrödinger equation

The meditation of quantum information
March 17th, 15:00-15:30, A 60 (Architekturgebäude) Technische Universität Berlin

The President Str. des 17. Juni 135

• THE THESIS AND HYPOTHESIS
The thesis:

- The Einstein field equation (EFE) can be directly linked to the Schrödinger equation (SE) by meditation of the quantity of quantum information and its units: qubits.
- One qubit is an “atom” both of Hilbert space and Minkovski space underlying correspondingly quantum mechanics and special relativity.
- Pseudo-Riemannian space of general relativity being “deformed” Minkowski space therefore consists of “deformed” qubits directly referring to the eventual “deformation” of Hilbert space.
Thus both equations can be interpreted as a two different particular cases of a more general equation referring to the quantity of quantum information (QI).

They can be represented as the transition from future to the past for a single qubit in two isomorphic form:

- SE: the normed superposition of two successive “axes” of Hilbert space
- EFE: a unit 3D ball

A few hypotheses to be ever proved are only formulated on the ground of the correspondence between the two equations in terms of quantum information or for a single qubit.
The underlying understanding of time, energy, and quantum information

<table>
<thead>
<tr>
<th>The flat case (SE, infinity, the axiom of choice):</th>
<th>The curved case (EFE, finiteness):</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Flat Case Diagram" /></td>
<td><img src="image2" alt="Curved Case Diagram" /></td>
</tr>
<tr>
<td><strong>Future</strong></td>
<td><strong>Future</strong></td>
</tr>
<tr>
<td><strong>Potential energy</strong></td>
<td><strong>Potential energy</strong></td>
</tr>
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<td><strong>Present</strong></td>
<td><strong>Present</strong></td>
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<tr>
<td><strong>Kinetic energy</strong></td>
<td><strong>Kinetic energy</strong></td>
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<tr>
<td><strong>Past</strong></td>
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<td><strong>Energy newly</strong></td>
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<td><img src="image3" alt="Energy Newly" /></td>
<td><img src="image4" alt="Energy Newly" /></td>
</tr>
<tr>
<td><img src="image5" alt="Energy Newly" /></td>
<td><img src="image6" alt="Energy Newly" /></td>
</tr>
</tbody>
</table>

\[ 1) = 0 \quad 2) = \]
The underlying understanding of time, energy, and quantum information

The flat case (SE, infinity, the axiom of choice):

- Future
- Potential energy

- Present
- Kinetic energy

\[ \equiv (SE) \]

The “curved” case (entanglement, infinity):

- Past
- Energy newly

\[ = ? \]
The main hypothesis

The underlying understanding of time, energy, and quantum information

The flat case (SE, infinity, the axiom of choice):
- Future Potential energy
- Present Kinetic energy
- Past Energy newly

The curved case (EFE, finiteness):

1) $i = 1$
2) $i = \infty$

\[ \sum_{i=1}^{\infty} \]
A sketch to prove the main hypothesis (1)

- The relative “curving” ($\Delta$) of a curved qubit to another can be represented by two curved qubits, the latter of which is described to the former.
- Each of them is equivalent to a state of pseudo-Riemannian space in a moment of time.
- The main hypothesis expresses the relation of one and the same kind of decomposition taken two times.
- Consequently, that kind of decomposition is sufficient to be discussed.
- It turns out to be equivalent to a principle of GR:
A sketch to prove the main hypothesis (2)

• In fact, the main hypothesis is equivalent to the premise of general relativity about the equality of inertial and gravitational mass. Indeed:

• The main hypothesis is a form of Fourier expansion if one considers the deformation of a qubit as an impulse in Minkowski space (or as a three-dimensional impulse): Then the sum bellow is just its Fourier expansion

• At last, that sum is isomorphic to an arbitrary trajectory in some potential field in Minkowski space since the entanglement should be a smooth function as both wave functions constituting it are smooth
A short comment of the above sketch

• However the member in EFE involving the cosmological constant \(\left({g_{\mu\nu}\Lambda}\right)\) means just that gravitational and inertial mass are not equal.

• As that constant was justified by Einstein (1918)’s “Mach’s principle”, this means that “Mach’s principle” had removed or generalized the equality of gravitational and inertial mass, which in turn is a generalization of the classical energy conservation to energy-momentum.

• The member of the cosmological constant added still one kind of energy without any clear origin.
The link between the generalizations of energy conservation in EFE and SE

• SE also generalizes the classical energy conservation adding the wave function and the member of its change in time (“\(i\hbar \frac{\partial}{\partial t} \Psi(r,t)\)”: 
• Indeed if \(\Psi(r,t)\) is a constant both in space and time SE is to reduce to energy conservation 
• However SE is fundamentally “flat” as well as Hilbert space and cannot involve any member analogical to the EFE generalization about energy-momentum
From SE to the case of entanglement

• Consequently the member \( \hat{\hbar} \frac{\partial}{\partial t} \Psi(r,t) \) in SE is what should correspond to the cosmological constant in EFE (though its member is multiplied by the metric tensor “\( g_{\mu\nu} \): “\( g_{\mu\nu}\Lambda\)”)

• One might suggest that entanglement will introduce some additional energy if the wave function “\( \Psi(r,t) \)” is substituted by its “curved” version: \( \hat{A}[\Psi(r,t)] \), where \( \hat{A} \) is not any self-adjoint operator in Hilbert space

• However the case of entanglement can take place only in relation to the “global” EFE rather than to the “local” EFE at a point because the local EFE corresponds to the usual SE
### Meaning | Temporal EM | Potential EM | Kinetic EM | Gravitational EM
---|---|---|---|---
EFE | $g_{\mu\nu}\Lambda$ | $R_{\mu\nu}$ | $\frac{1}{2} g_{\mu\nu} R$ | $\frac{8\pi G}{c^4} T_{\mu\nu}$
SE | $i\hbar \frac{\partial}{\partial t} \Psi(r, t) \cdot V(r, t) \cdot \Psi(r, t)$ | $-\frac{\hbar^2}{\mu} \nabla^2 \Psi(r, t)$ | 0

### QI correspondence
- Bits: Q in the past
- Potential Q
- Kinetic Q
- Deformed Q
- Entangled Q

### Equations
\[ EFE: \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu} \]
\[ SE: \quad i\hbar \frac{\partial}{\partial t} \Psi(r, t) = \left[ -\frac{\hbar^2}{\mu} \nabla^2 + V(r, t) \right] \Psi(r, t) \]
(SE for a single non-relativistic particle)
The emphasized terms (in red):

They are new and mean:

- **“Temporal EM”**: EM of a leap unit (or a bit) along the geodesic line of space-time

- **“Potential Q”**: a qubit in the physical dimension of action, which is equivalent to a qubit in the physical dimension of space-time for a unit of energy-momentum (a tentative reference body or “material point”)

- **“Kinetic Q”**: a qubit in the physical dimension of the energy-momentum of mechanical motion

- **“Deformed Q”**: the Ricci tensor of a “kinetic Q”
The other notations (EFE)

- $R_{\mu\nu}$, Ricci curvature tensor
- $g_{\mu\nu}$, metric tensor
- $R$, scalar curvature
- $\Lambda$, cosmological constant
- $G$, Newton's gravitational constant
- $c$, speed of light
- $T_{\mu\nu}$, stress–energy tensor
The other notations (SE):

- $i$, imaginary unit
- $\hbar$, the Planck constant divided by $2\pi$
- $\Psi(r, t)$, wave function
- $V(r, t)$, potential energy
- $\mu$, reduced mass
- $\nabla^2$, Laplacian
- $r$, space vector
- $t$, time
The sense of the correspondence

• The Ricci curvature tensor, $R_{\mu\nu}$, is a quantity designating the deformation of a unit ball

• $\frac{1}{2} g_{\mu\nu} R$ is the “half” of the Ricci curvature tensor

• The subtraction of these is the “other half” of the Ricci curvature tensor

• EFE states that this “other half” is equal to energy-momentum eventually corrected by the cosmological constant member

• *SE states the analogical about the “flat” case*
• Schrödinger equation is interpreting as generalizing energy conservation to the past, present, and future moments of time rather than only to present and future moments as this does the analogical law in classical mechanics

• The equation suggests the proportionality (or even equality if the units are relevantly chosen) of the quantities of both quantum and classical information and energy therefore being a (quantum) information analog of Einstein’s famous equality of mass and energy ("E=mc^2")
A few main arguments

1. The three of the EFE members (sells 2.3, 2.4, and 2.5 of the table) are representable as Ricci tensors interpretable as the change of the volume of a ball in pseudo-Riemannian space in comparison to a ball in the three-dimensional Euclidean space (3D).

2. Any wave function in SE can be represented as a series of qubits, which are equivalent to balls in 3D, in which two points are chosen: the one within it, the other on its surface.
3. The member of EFE containing the cosmological constant corresponds to the partial time derivative of the wave function in SE (column 2 of the table).

- This involves the energetic equality of a bit and a qubit according to the quantum-information interpretation of SE.

- The zero cosmological constant corresponds to the time-independent SE.
4. The member of EFE, which is the gravitational energy-momentum tensor, corresponds to zero in SE as it express that energy-momentum, which is a result of the space-time deformation (column 5).

5. SE represents the case of zero space-time deformation, EFE adds corresponding members being due to the deformation itself (row 3).
A “footnote” about quantum correlations

The flat case (the axiom of choice):

The future and present in terms of the past: well-ordered

The case of correlations (Kochen-Specker theorem):

Future and present in terms of future: unorderable
The “footnote” continuous:

The correlations are certain

The case of correlations\(\star\) (Kochen-Specker theorem):

Entanglement: (the correlations in terms of the past

The correlations are uncertain
• INTERPRETATIONS
About “relativity” in general relativity

• One possible philosophical sense of relativity is that the physical quantities are relations between reference frames rather than their properties.

• Nevertheless, if one formulates relativity both special and general in terms of Minkowski or pseudo-Riemannian space, any relation of reference frames turns out to be a property of the corresponding space.

• The “relation” and “property” themselves as well as relativity turn out to be also relative in turn.
About “relativity” in quantum mechanics

• Quantum mechanics is formulated initially in terms of properties, which are “points” in Hilbert space.

• However, all experimentally corroborated phenomena of entanglement mean that it can be not less discussed in terms of relations.

• Thus both quantum mechanics and relativity turn out to share the generalized relativity of relation and property.

• Even more, their “spaces of properties” relate to each other as the two “Fourier modifications” of one and the same generalized space in order to realize the invariance of time (continuity) and frequency or energy (discreteness).
Quantum invariance and the relativity of relation and property

- One can call that invariance of time (continuity) and frequency or energy (discreteness) “quantum invariance” shortly.
- Quantum invariance and the relativity of relation and property are equivalent to each other and even to wave-particle duality as follows:

<table>
<thead>
<tr>
<th>Continuity – discreteness</th>
<th>Relation – property</th>
<th>Wave – particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Relation (parts)</td>
<td>Particle</td>
</tr>
<tr>
<td>Frequency (energy)</td>
<td>Property (a whole)</td>
<td>Wave</td>
</tr>
</tbody>
</table>
The Standard model and that relativity

• The Standard model itself can be discussed in terms of that relativity first of all in order to be elucidated its relation to gravity remained out of it.

• If general relativity is the theory of gravity, one can deduce that the Standard model and general relativity share the same duality as any pair of the previous table.

• This means that gravity is not more than an force (interaction) equivalent to the rest three:

• The former is the continuous (classical) hypostasis, the latters are the discrete (quantum) one of one and the same.
The “fixed point” of electromagnetism

• Electromagnetism turns out to be shared by both the Standard model as the symmetry U(1) and general relativity as the particular, borderline “flat” case of Minkowski space.

• Even more, it is shared by quantum information as “free qubit”, i.e. as the sell of qubit, in which is not yet “recorded” any value.

• Consequently, electromagnetism is to be discussed as the fixed point of quantum invariance after it is that for the two hypostases of general relativity and the Standard model.
The ridiculous equivalence of general relativity and the Standard model

• If one gaze carefully at that equivalence of general relativity and the Standard model, it turns out to abound in corollaries ridiculous to common sense:

• For example, the quantum viewpoint is valid to an external observer, and the relativity point of view is valid to an internal observer, but both are equivalent to each other after entering / leaving the system:

• As if one leaves our world, it would be transformed into a quantum one, as if one enters the quantum world, it would look our as ours
The Standard model as a certain and thus privileged reference frame

The “Big Bang” refers always to the past: Thus it can be seen whenever only “outside”

Its privileged counterpart in pseudo-Riemannian space is the space-time position of the observer “here and now” wherever it is:

All possible “here and now” share one and the same tangent Minkowski space determined by the constant of the light speed, and by one and the same position of the Big Bang, which is forever in the past and thus outside
The “Big Bang” privileges a reference frame, and it is consistent to the Standard model. Nevertheless, the positions inside distinguish from each other by different curvature and therefore by their different wave function. Even more, one can suggest that the Standard model is exactly equivalent to the description to a single inertial reference frame, that of the “Big Bang”:

Whether as a common tangent to all possible positions in space-time
Or as a common limit of converging of all possible wave functions
Indeed the Standard model uses the intersection of the tensor product of the three fundamental symmetries $[U(1)] \times [SU(2)] \times [SU(3)]$:

However this is necessary in order to construct the dual counterpart of an inertial reference frame equivalent to a single qubit, to which all possible wave functions in our universe converge.

All this seems to be the only, which is able to be consistent with the discussed equivalence of general relativity and the Standard model:

That equivalence means the rather trivial coincidence of a manifold and its tangent space at any point.
• CONCLUSIONS:
That equation, of which both EFE and SE are particular cases

• Both turn out to be the two rather equivalent sides of a Fourier-like transformation

• Thus the cherished generalized equation seems to coincide with both EFE and SE

• Nevertheless a certain fundamental difference between them should be kept:

• SE refers to the “flat” case of the tangent space in any point of a certain manifold described by EFE
SE as the EFE at a point

Thus SE refers to the representation into an infinite sum (series) at a point of a manifold described by EFE. Roughly speaking, SE and EFE coincide only locally but everywhere, i.e., at any point. This results both in coinciding the way for the present to be described and in distinguishing the ways for the past and future to be represented:

“Flatly” by SE, so a qubit can describe and refers equally well to each modus of time: This is its sense.

“Curvedly” by EFE, so that the past and future are described as present moments however “curved” to the real present, which is always “flat”.

They describe time differently, but equivalently.
The global EFE as the SE after entanglement

• Though locally coinciding, EFE and SE differs from each other globally:

• Then the following problem appears: How one or both of them to be generalized in order to coincide globally, too

• The concept of quantum gravity suggests that EFE is what needs generalization by relevant quantization, in which nobody has managed, though

• The present consideration should demonstrate that the SE generalization is both much easier and intuitively more convincing
“Relative quant” vs. “quantum gravity”

- Once SE has been generalized, perhaps EFE will be generalized reciprocally and mathematically, too.
- One can coin the term of relative quant as the approach opposite to quantum gravity.
- Its essence is the manifold described by EFE to be represented by the set of tangent spaces smoothly passing into each other.
- One needs only describing SEs in two arbitrarily rotated Hilbert spaces as tangent spaces at two arbitrarily points of pseudo-Riemannian space, i.e. SE generalized about entanglement.
Quant of entanglement as relative quant

• Consequently the concept of relative quant is to be interpreted as the concept of entanglement applied to general relativity and thus to gravity

• However the qubit is also the quant of entanglement as well as the common quant of Hilbert, Minkowski and even pseudo-Riemannian space

• Indeed a qubit can express the relative rotation of two axes of the same “n” in two arbitrarily rotated Hilbert space exactly as well as the relation of two successive axes in a single Hilbert space
A deformed qubit as a deformed unit vector of the basis on Hilbert space

• One can think of a qubit as follows: its unit vector is an “empty” unit ball (\(e^{i\omega} = e^{in\omega}\)), in which two points are chosen (“recorded”), the one of which on the surface, the other within the ball.

• The same ball can be transformed arbitrarily but only smoothly (conformally):

• It will be also a deformed “relative quant” at a point in pseudo-Riemannian space, and after this represented as a wave function in the tangent Hilbert space of the common entangled system.
Involving Fock space

- Fock space serves to describe a collection of an arbitrary number of identical particles whether fermions or bosons, each of which corresponding to a single Hilbert space
- Informally, it is the infinite sum of the tensor product of $i (i = 0, 1, \ldots, \infty)$ Hilbert spaces
- Formally: $F_{\nu}(H) = \bigoplus_{n=0}^{\infty} S_{\nu} H^{\otimes n}$
- Then pseudo-Riemannian space can be described by infinitely many entangled Hilbert spaces by means of Fock space as an entangled state of infinitely many identical (bosonic) particles
Notations:

\( F(H) \), Fock space on Hilbert space, \( H \)

\( \nu \), a parameter of two values, the one for bosonic, the other for fermionic ensemble

\( S_\nu \), an operator symmetrizing or anti-symmetrizing the tensor involution, which follows, correspondingly for bosons or fermions

\( H \otimes n \), the \( n^{th} \) tensor involution of \( H \)
Entanglement by Fock space and the global EFE

• Indeed the pseudo-Riemannian manifold can be represented by infinitely many points of it.
• There are infinitely many different wave functions corresponding one-to-one to each point and a tangent Hilbert space, to which the corresponding wave function belongs.
• However, these Hilbert spaces are rotated to each other in general.
• Nevertheless, one can utilize Fock space where the Hilbert spaces are not entangled to represent the entangled case as above.
The main conclusion

• The global EFE is equivalent to an infinite set of SEs, each of which is equivalent to the local EFE at as infinitely many points as points are necessary to be exhaustedly described the only manifold corresponding to the global EFE.

• In other words, that SE, which corresponds to the global EFE, can be written down as a single equation in Fock space for infinitely many “bosons” describing exhaustedly the pseudo-Riemannian manifold.
How can one prove that?

- Entanglement of two Hilbert spaces can be equivalently represented by an infinite sum in Fock space.
- Indeed one can think of Fock space as that generalization of Hilbert space where the complex coefficients $\{C_i\}, i = 1, ..., \infty$, of any element (vector), which are constants, are generalized to arbitrary functions of Hilbert space $\{\Psi_i\}$.
- This implies the necessity of an infinite set of points (i.e. members of Fock space or “bosons”) to be represented the pseudo-Riemannian manifold exhaustedly.
Furthermore ...

• The curved manifold of entanglement can be better and better approximated adding new and new, higher and higher nonzero members in Fock space.

• Thus coincidence is possible only as a limit converging for infinitely many members of Fock space.

• Thus Fock space in infinity can represent a generalized leap in probability distribution: However if the limit as above exists, that leap will be equivalent to a single smooth probability distribution corresponding to a single and thereupon “flat” Hilbert space of the entangled system as a whole.
Consequently:

1. The global EFE represent a generalized “quantum” leap, that in probability distribution, and equivalent to some entanglement just as the local one

2. Nevertheless, this is observable only “inside”: If the corresponding relation is reduced to the equivalent property, which means that the entangled system is observed as a whole, i.e. “outside”, there will not be both entanglement and gravity

3. Thus the Standard model being referred to a single quantum system observed “outside” can involve neither gravity nor entanglement: they exist only in the present, but not in the past and future
References:


Vielen Dank für Ihre Aufmerksamkeit!
Leiten Sie Ihre Fragen und Kommentare!